ABSTRACTS OF ARTICLES DEPOSITED AT VINITI*

STOCHASTIC HEAT-CONDUCTION AND THERMOELASTICITY PROBLEM FOR A HALF-SPACE

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The half-space $x \geq 0$, in thermal contact with the external medium, is investigated. Convective heat transfer takes place between the surface $x = 0$ of the half-space and the external medium according to Newton's law

$$\frac{\partial T}{\partial x} - \frac{\alpha}{\lambda_q}(T - \theta) = 0.$$

The temperature θ of the external medium is a stochastic time function [1]. The initial temperature of the half-space and the medium next to the surface $x = 0$ is T_0.

The dynamic thermal stresses induced in the half-space by the nonstationary stochastic temperature field are determined for zero stresses at the surface and homogeneous initial conditions [2].

For the case in which the temperature θ of the external medium is described by the canonical series

$$\theta(t) = \langle \theta \rangle + \sum_j \xi_j \theta_j(t),$$

the solutions of the heat-conduction and thermoelasticity boundary-value problems are represented with unit probability in the canonical forms

$$T(x, t) = \langle T \rangle + \sum_j \xi_j T_j(x, t);$$

$$\sigma_x(x, t) = \langle \sigma_x \rangle + \sum_j \xi_j \sigma_{xj}(x, t).$$

Here ξ_j denotes independent stochastic variables with zero mathematical expectation; θ_j denotes deterministic time functions; and $\langle T \rangle$, $\langle \sigma_x \rangle$, T_j, σ_{xj} are the solutions of the corresponding deterministic problems.

The variances for the temperature field and thermal stresses of the half-space have the form

$$D_T(x, t) = \sum_j D_j |T_j(x, t)|^2; \quad D_\sigma(x, t) = \sum_j D_j |\sigma_{xj}(x, t)|^2,$$

where D_j is the variance of the stochastic variable ξ_j.

Let ξ_j represent stochastic variables with a normal distribution and zero expectation. Then on the basis of the expressions for the expectations and variances of the temperature field and thermal stresses at every fixed point of the half-space we determine the probability that they will fall in specified intervals. A numerical example is discussed.

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APPLICATION OF THE THERMAL POTENTIALS METHOD TO SOLVE A NON-STATIONARY HEAT-CONDUCTION PROBLEM FOR A TWO-LAYERED HALF-SPACE

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A boundary-value problem of heat conduction for an unlimited plate in thermal contact with a half-space with conditions of the third kind on its outer boundary reduces to a Volterra-type integral equation of the second kind for the density of the thermal potential:

$$\varphi(\tau)-\varepsilon\int_0^\tau \varphi(\tau')f_0(\beta,\ \tau-\tau')\,d\tau'=A(\tau)\left[u(\tau)-\int_0^\tau \varphi(\tau')(\varphi_1(0,\ \tau-\tau')+\varepsilon f_1(\beta,\ \tau-\tau'))\,d\tau'\right],$$

where

$$f_p(\beta,\ \tau)=\frac{1}{\sqrt{\pi}}\tau^{p-3/2}\beta^{1-p}\exp(-\beta^2/\tau),\ p=0.1;$$

$A(\tau)$ is the dimensionless coefficient of heat elimination; $u(\tau)$ is the temperature of the outer medium; β is the dimensionless plate thickness; and ε is a factor taking account of the condition of thermal contact between the plate and the half-space. For a heat-insulated plate $\varepsilon = 0$.

By using special quadrature formulas which are exact when $\varphi(\tau)$ are represented in the form of a broken line, the integral equation reduces to a system of algebraic equations with a triangular coefficient matrix to determine the values of the thermal potential density at the nodes of the lines.

The temperature of the outer plate surface is found simultaneously with the solution of the integral equation. The temperatures of the inner points of the two-layered half-space are calculated by using quadrature formulas of the same kind.

Since the coefficients of the quadrature formulas determined by using recurrence relations are independent of $A(\tau)$ and $u(\tau)$, the proposed method is especially convenient for performing a large series of computations with different boundary conditions for the same structure.

The boundary-value problem with boundary conditions of the second kind is solved analogously.

In contrast to the mesh method, the method proposed is very much simpler, since the desired function depends only one one variable and imposes no rigid constraints on the magnitude of the time breakdown interval.

An example is presented of the temperature field computations for a two-layered half-space in the well-known particular case of constant heat flux at the boundary. Comparison of the results exhibited good accuracy for the method proposed.

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SOLUTION OF THE UNSTEADY HEAT-CONDUCTION EQUATION IN CONTACTING BODIES OF COMPLEX SHAPE WITH BOUNDARY CONDITIONS OF THE THIRD KIND

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We consider the solution of the unsteady heat-conduction equation

$$\Delta T_i(p, t) + \frac{Q_i(p, t)}{\varkappa_i} = \frac{1}{a_i}\,\frac{\partial T_i(p, t)}{\partial t}, \; p \in V_i \tag{1}$$

in a system consisting of N homogeneous contacting bodies of complex shape with the initial conditions

$$T_i(p, 0) = f_i(p), \; p \in V_i, \tag{2}$$

and the boundary conditions

$$\varkappa_i \frac{\partial T_i(p, t)}{\partial n} + \alpha_i T_i(p, t) = \varphi_i(p, t), \; p \in S_{i0}, \tag{3}$$

for surfaces which are in actual thermal contact

$$\varkappa_i \frac{\partial T_i(p, t)}{\partial n} = \varkappa_k \frac{\partial T_k(p, t)}{\partial n}, \; p \in S_{ik}, \tag{4}$$

$$-\varkappa_i \frac{\partial T_i(p, t)}{\partial n} = \frac{1}{R_{ik}}\,[T_i(p, t) - T_k(p, t)], \; p \in S_{ik}. \tag{5}$$

In the first part of the paper the boundary value problem (1)-(4) is solved by reducing it to the problem of solving a system of linear algebraic equations by taking the Laplace transform, using the method of partial domains, the apparatus of R functions for finding the solution in the i-th domain, and one of the projection methods.

Using the proposed method the temperature distribution in two contacting bodies having the shape of a triangle and four circles is constructed, assuming actual thermal contact between the contacting surfaces.

The second part of the paper is devoted to an examination of the possibility of using the Bubnov–Galerkin method to reduce the problem to that of solving a system of linear integral equations. A system of integral equations is obtained if the solution in a partial domain can be constructed by the separation of variables, the apparatus of Green's functions, or the method of finite integral transforms. Using the proposed method the temperature distribution in a two-layer plate with known functions on the surfaces is found for boundary conditions of the first and third kinds.

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